Application of the Markov Chains in the MTPL Insurance on the Example of the Polish Market

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RESUMEN: Las claúsulas bonus-malus son uno de los tipos de tarificación en los seguros de vehículos. En este artículo se analiza el impacto de los cambios en las reglas de paso entre las distintas clases, así como la eficiencia del aumento del número de éstas. Para evaluar la eficacia de las tarifas se han utilizado medidas estocásticas basadas en la teoría de Cadenas de Markov. A modo de ejemplo, se ha evaluado la eficiencia del sistema de tarifas de bonus-malus de la mayor compañía de seguros polaca, PZU. Los resultados del análisis son comparados con la eficiencia de las tarifas construidas en este estudio y con otras reglas de transición entre las clases de bonus-malus.

ABSTRACT: Bonus-malus systems are one of the tariffication stages in car insurance. In the paper the impact of changing rules of passing between classes and increasing the number of classes on tariffication bonus-malus systems efficiency are analysed. To assess the tariff effectiveness the stochastic measures based on the theory of Markov chains were applied. As an example the tariff efficiency of bonus-malus system of the largest Polish insurance company, PZU, was used. The results of analysis were compared with the tariff efficiency constructed in the study on systems with a larger number of classes and other rules of the transition between the bonus-malus classes.

Keywords:

MTPL insurance, effectiveness measures of bonus-malus systems; Markov chains.

Subject area: actuarial mathematics.

1. INTRODUCTION

To compete in local markets, insurance companies use different means with premium rate which are the most important ingredient of their strategies. The insurer's main task is to adjust the premium amount appropriately to the level of risk represented by drivers. The risk is understood here as the insurer's expected loss, which depends on the number and amount of losses.

Their financial stability should be guaranteed by premium rates. In most of European Union countries insurers are able to set their own tariffs. The premium in car insurance is calculated in a two-stage process. The first stage called – *a priori* ratemaking – involves calculation of a basic premium on the grounds of observable risk factors such as the car engine displacement, the place of residence or horsepower of the engine. The ratemaking variables may differ from country to country within European Union (EU). In Poland the car registration region and the engine displacement are used. The second stage– *a posteriori* –is built on the claim record of the insured and involves discounts or surcharges that are applied to the basic premium. In an *a posteriori* stage of defining a tariff policy in car insurance, bonus-malus systems are used. Each bonus-malus system is comprised of classes such that each of them has a different bonus-malus coefficient, which is the ratio of the written premium. Rules of transition are used to assign drivers to classes [Lemaire (1995), Hossack (1983)].

Since different bonus-malus systems are applicable, they can be, first of all, modelled differently and, secondly, it becomes necessary to compare the effectiveness of systems.

The theory of Markov chains can be used for the analyzing and forecasting of many economic processes, including claims history in car insurance. Under certain assumptions the system of increases and reductions, can be modeled by use of Markov chains. The contribution rate, the number of classes of the system and of the transitions between classes affect the effectiveness of the bonus-malus system.

The aim of the paper is to present a model of the premium increases and reductions system for loss-free driving in Motor TPL preserving Markov chains, to determine effectiveness measures of bonus-malus systems, and evaluate the influence of changes in transition rules and increase of the number of classes on effectiveness of a bonus-malus system. The other three systems are the modification of the PZU system consisting of increasing the number of classes and changing the rules of transition between classes. Each system was evaluated using stochastic efficiency measures. It was assumed that the distribution of the number of claims is negative binomial with parameters estimated on the basis of data from the Polish market.

2. THE MARKOV CHAINS IN BONUS-MALUS SYSTEM

The following assumptions have been accepted for modeling the premium increases and reductions system in Motor TPL Insurance by means of THE Markov chains:

1) A fixed group of drivers (the policyholders) divided into risk classes called tariff classes;
2) The number of tariff classes is finite and amounts to s. $S=\{1,2,...,s\}$ will denote a set of tariff class numbers. Let us accept that class $j=1$ is burdened with the highest premium increases and class $j=s$ with the biggest reductions;
3) The insured's classification in class i in a given year is dependent upon the class, from the previous year and the number of losses caused in the previous year. It could be added that drivers without a loss history will be classified in the starting class. Let C_t be the random variable describing the class to which a given driver belongs in the period $(t-1,t]$, with C_{i_0} being the first (starting) class;
4) The number of losses in a given year for any driver in a given class is random variable K with probability distribution being known and constant over time. The amount of losses caused by an individual driver is random variable Y. Variables K and Y are independent variables. Random variable X is the total value of losses declared within any one time period, that is, during one year;
5) Premium b_i, $i = 1,...,s$ is attributable to each *i-th* class.

Let $T_k(i) = j$ denote the transition of the driver form the class i to the class j, if the k damages in one year were caused, where $T : S \to S$, $S = \{1,2,...,s\}$ ($i, j \in S$, $k = 0,1,2,...$). So, defined function is called the transition function whereas transition rules can be expressed as k binary matrices:

$$\mathbf{T}(k) = [t_{ij}(k)] = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1s} \\ \vdots & \vdots & \ddots & \vdots \\ t_{s1} & t_{s2} & \cdots & t_{ss} \end{bmatrix}, \tag{1}$$

where $t_{ij}^{(k)} = t_{ij}(k) = \begin{cases} 1 \text{ dla } T_k(i) = j \\ 0 \text{ dla } T_k(i) \neq j \end{cases}$ for $i, j \in S$, $k = 0,1,2,...$

The bonus-malus system model for one policyholder with a constant damage intensity parameter $\lambda > 0$ is a homogenous Markov chain $\{C_t\}_{t\in N}$ with the state space $S = \{1,2,...,s\}$, the transition matrix of the form:

$$\mathbf{M}(\lambda) = \sum_{k=0}^{\infty} p_k(\lambda)\mathbf{T}_k \tag{2}$$

and the probability of transfer of the policyholder $p_{ij}(\lambda)$ from the tariff class C_i to the tariff class C_j in one year equals to:

$$p_{ij}(\lambda) = \sum_{k=0}^{\infty} p_k(\lambda) t_{ij}^{(k)}, \tag{3}$$

where $p_k(\lambda)$ is the probability of k claims in one year for the given policyholder.

By modeling bonus-malus systems with the use of Markov chains the main problem in is finding the stationary (equilibrium) distribution of the chain under the assumption of its homogeneity and ergodicity. For each irreducible ergodic Markov chain exists one equilibrium distribution of the form:

$$\mathbf{a}(\lambda) = [a_1(\lambda), \ldots, a_s(\lambda)], \tag{4}$$

where $a_j(\lambda) = \lim_{n\to\infty} p_{ij}^n(\lambda)$ and $p_{ij}^n(\lambda)$ is the transition probability of the policyholder in n years from the C_i class to the C_j class. The equilibrium distribution can be found by equation:

$$a_j(\lambda) = \sum_{i=1}^{s} a_i(\lambda) p_{ij}(\lambda), \quad j = 1, \ldots, s, \tag{5}$$

where $\sum_{j=1}^{s} a_j(\lambda) = 1$. The element $a_j(\lambda)$ of the vector $\mathbf{a}(\lambda)$ denotes the fraction of the policyholder that belong to the class C_j after reaching the equilibrium state by the system or the probability that the insured belongs to the class C_j in n periods, when the number of periods convergent to infinity. On the assumptions given above the vector $\mathbf{a}(\lambda)$ can be calculated as a normalized left Eigen vector of the matrix $\mathbf{M}$ transition.

The application of Markov chains theory to bonus-malus systems modelling can be found in papers by Bonsdorff (1992), Lemaire (1985), (1995), Mahmoudvand et al (2013), Niemiec (2007).

3. MEASURES OF TARIFF EFFECTIVENESS OF BONUS-MALUS SYSTEMS

There are many effectiveness measures of bonus-malus systems. The Loimaranta effectiveness $\eta(\lambda)$ is most commonly used measure evaluating a bonus-malus system, which is the elasticity of the mean stationary premium with respect to the claim frequency [Lemaire (1995)], given by:

$$\eta(\lambda) = \frac{dB(\lambda)}{B(\lambda)} \Big/ \frac{d\lambda}{\lambda}, \tag{6}$$

where the expected stationary premium for the single period, after the system reaches the state of equilibrium equals to:

$$B(\lambda) = \sum_{i=1}^{s} a_j(\lambda) \cdot b_j \,. \tag{7}$$

So, defined by Loimaranta efficiency is the flexibility of an average premium B (λ) with respect to the level of risk λ and allows to assess the extent to which drivers are evaluated by the system. In perfect condition should be an increasing function λ such that $\eta(\lambda) = 1$. Thus, a good bonus-malus system is a system with the flexibility equal to one in which the premium rates move in the same way as the intensity of damages of the policyholder.

The values of the elasticity function (6) depend on the risk parameter λ. That is the reason why are the elasticity functions often compared or, as an alternative, the so called joint elasticity is calculated by numerically solving the integral:

$$\eta = \int_0^\infty \eta(\lambda)\pi(\lambda)d\lambda. \tag{8}$$

Lemaire (1985) proposed a measure of bonus-malus system effectiveness which is called the relative stationary average level:

$$RSAL(\lambda) = \frac{B(\lambda) - \min_j(b_j)}{\max_j(b_j) - \min_j(b_j)}. \tag{9}$$

The measure given by the formula (9) informs about the relative position of the policyholder with the average claim record in case when the value 0 is assigned to the lowest premium and the value 1 to the highest one. This measure does not have an optimal value which would be acknowledged in the literature. According to the same author, in the ideal case the value of this indicator should equal 0.5 for the average claim record. Low values of *RSAL* indicate that the system is imbalanced and over time majority of the policyholders are going to belong to the classes with highest discounts. Large values of this measure correspond to the uniform distribution of the policyholders among the BMS classes.

4. PRACTICAL EXAMPLE OF THE MARKOV CHAINS'APPLICATION

Tables 1-4 present the Bonus-Malus PZU system and systems of: BMS 1, BMS 2 and BMS 3 proposed in the analysis of bonus-malus.

BM class	Premium rate	Number of claims per year

	[%]	**0**	**1 and more**
1	200	2	1
2	150	3	1
3	130	4	1
4	115	5	2
5	100	6	3
6	90	7	4
7	80	8	5
8	80	9	6
9	70	10	7
10	60	11	8
11	50	12	9
12	50	13	10
13	40	14	11

Table 1 BMS PZU

BM class	**Premium rate [%]**	**Number of claims per year**		
		0	**1**	**2 and more**
1	300	2	1	1
2	260	3	1	1
3	200	4	2	1
4	150	5	3	2
5	115	6	4	3
6	100	7	5	4
7	95	8	6	5
8	90	9	7	6
9	85	10	8	7
10	80	11	9	8
11	75	12	10	9
12	70	13	11	10
13	65	14	12	11
14	60	15	13	12
15	55	16	14	13
16	50	17	15	14

17	45	18	16	15
18	40	18	17	16

Table 2 BMS 1

BM class	Premium rate [%]	Number of claims per year		
		0	1	2 and more
1	300	2	1	1
2	260	3	1	1
3	200	4	1	1
4	150	5	2	1
5	115	6	3	2
6	100	7	4	3
7	95	8	5	4
8	90	9	6	5
9	85	10	7	6
10	80	11	8	7
11	75	12	9	8
12	70	13	10	9
13	65	14	11	10
14	60	15	12	11
15	55	16	13	12
16	50	17	14	13
17	45	18	15	14
18	40	18	16	15

Table 3 BMS 2

BM class	Premium rate [%]	Number of claims per year		
		0	1	2 and more
1	300	2	1	1
2	260	3	1	1
3	200	4	1	1
4	150	5	1	1
5	115	6	1	1

6	100	7	1	1
7	95	8	6	1
8	90	9	6	1
9	85	10	6	1
10	80	11	6	1
11	75	12	6	1
12	70	13	6	1
13	65	14	6	1
14	60	15	6	1
15	55	16	6	1
16	50	17	6	1
17	45	18	6	1
18	40	18	6	1

Table 4 BMS 3

In each of the analyzed systems, the lack of damage in a given year is being rewarded moving on to the next grade with more discount in next year.

The BM PZU system has 13 classes, including 4 surcharge classes with the maximum surcharge of 200%. In the PZU system a claim in a given year entails an increase of a number of degrees in the bonus-malus system and transfer to the former class (by one class upwards in the Table 1) with a higher premium at the renewal of the policy.

Other bonus-malus systems proposed in the study have 18 classes, so more than the BM PZU system. There are more surcharge classes and the maximum surcharge is of 300%. BMS 1 is system where one claim corresponds to the increase on a bonus-malus scale by one class and two damage claims or more entail the transfer two classes upwards.

BMS 2 is more restrictive for the insured. In case of one damage the insured moves two class upwards in Table 3. Three damage claims result in the transition three classes upwards.

The BMS 3 system is the most restrictive. Here one damage in a year implies loss of every discount and transition to the starting class i.e. the fifth class (BM 5) in Table 4. In case of a driver with no discount one claim results in a transfer to the first class with maximum surcharge (BM 1). In this system, for two or more damages the policyholder is also given the maximum premium surcharge, so it again goes to the first class (BM 1) in Table 4.

5. THE EVALUATION OF BONUS-MALUS SYSTEMS EFFICIENCY

In the paper we assumed that the number of damages has the negative binomial distribution with the intensity parameter $\lambda = 0{,}043$, which is similar to the damage intensity parameter for the whole car insurance market in Poland. The negative binomial distribution is used in majority of literature regarding models of the number of damages in car insurance [Ibiwoye et al (2011), Lemaire (1985)].

The values of the measures of effectiveness for the bonus-malus systems are presented in Table 5 and in Figure 1.

	$\eta($	η	$B(\lambda)$	$RSAL(\lambda)$
PZU	0, 128164	0,126651	0,410854	0,006783
BMS 1	0, 057002	0,056328	0,402573	0,000990
BMS 2	0, 124499	0,123028	0,408313	0,003197
BMS 3	0, 854902	0,844805	0,683036	0,108860

Table 5 Values of measures of effectiveness of bonus-malus systems

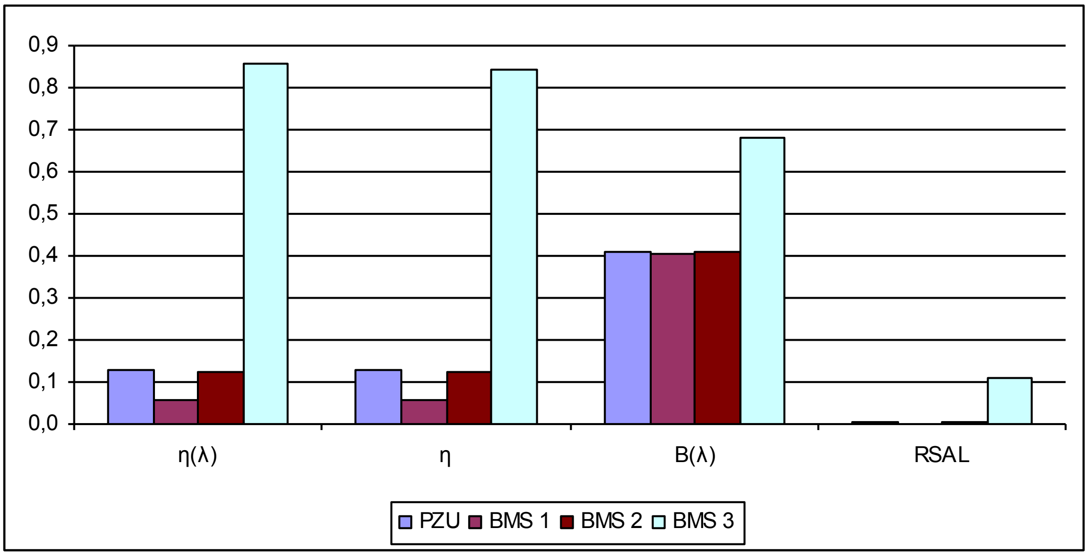

Figure 1 Values of measures of effectiveness of bonus-malus systems

Assessing the tariff effectiveness of BMS 1, BMS 2 and BMS 3 systems it can be stressed that the introduction of bonus-malus system more restrictive to drivers, causing damage, with the same number of classes, increases the tariff efficiency of the system. This confirms the results of previously conducted studies that evaluated the tariff effectiveness of the system BM PZU when changing the rules of the transition of policyholder between classes [Szymańska and Małecka (2013)]. Increasing the number of classes of the system does not always improve the tariff efficiency, as exemplified by the BM PZU and BMS 1 systems. The Bonus-Malus PZU, BMS 1 and BMS 2 systems are characterized by low tariff efficiency. So, the policies will be focused in the last class, which will cause an imbalance of the system. The bonus-malus system BMS 3 will better classify the policyholder, the expected fixed premium will be higher, and the focus of policies in discount classes will be smaller.

6. CONCLUSION

Summarizing, the rules of transition between bonus-malus classes directly affect the probability of move in a Markov chain, which makes changes of the tariff efficiency of bonus-malus system. With the same number of classes the more stringent are the rules of pass, the higher the efficiency of the bonus-malus system. Such a system better adjusts the class to the frequency of damage and policies do not focus at discounts classes. Increasing the number of classes does not always improve the tariff efficiency and may even lower it.

In Poland, nowadays, due to the "price war" in the MTPL insurance market it is not possible to introduce too stringent bonus-malus systems. As a result, the rates for bonus-malus systems are too mild for the policyholders causing higher than average number of damages per year. Premiums are too low, and technical result is negative in the insurance liability group.

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