Cournot equilibria for socially responsible firms in an uncertain environment.

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ABSTRACT

This paper considers firms which compete under Cournot assumptions and incorporate social responsibility to the evaluation of their results. In our model a socially responsible firm is one which takes into account not only its profits, but also it internalizes its own share of externality and is sensitive to consumer surplus.

The analysis of the equilibria to which the firms will eventually arrive is addressed in a framework where the results of the strategic decisions of the firms depend on a future uncertain event and no information about the probability distribution is available.

Keywords: Pareto equilibria, Cournot games, Uncertainty, Attitude to risk.
JEL classification: D43, D81, L10.

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RESUMEN

En este trabajo se analiza el efecto de la inclusión de objetivos de responsabilidad social en un modelo de empresas que compiten bajo los supuestos de Cournot. En nuestro modelo una empresa socialmente responsable es aquella que no solo tiene en cuenta sus beneficios, sino que también tiene en cuenta las externalidades positivas generadas por el excedente del consumidor.

El análisis de los equilibrios a los que pueden llegar las empresas se realiza en un contexto de incertidumbre. Los resultados de las decisiones estratégicas de las empresas dependen de la realización de un escenario futuro y no se dispone de información sobre las probabilidades de ocurrencia de los posibles escenarios.

Palabras clave: Equilibrios de Pareto, Juegos de Cournot , Incertidumbre, Actitud ante el riesgo, Responsabilidad social.

JEL classification: D43, D81, L10.

1 Introduction

In recent years, consumers have become increasingly aware of the role firms play within the social context. For that reason, firms now increasingly try to become socially responsible. At the same time, the features of Corporate Social Responsibility (CSR) and its impact on firm performance, especially in the field of management sciences and economics of organizations, has been receiving considerable attention in the academic community, from the CSR construct in the 1950s (Bowen, 1953) to empirical investigations on the relationship between CSR and corporate financial performance (Margolis and Walsh, 2001) and, then, to formal modeling of CSR (Baron, 2001, 2007; Calveras et al., 2007; Giovanni and Giacinta, 2007). A review of the theoretical and empirical economic literature on CSR behaviors is Crifo and Forget (2013). Another interesting review is Kitzmueller and Shimshack (2012), where the synthesis of diverse strands of the expanding CSR literature is presented.

One way to analyze the effects of strategic CSR is to introduce into the utility function of the social firm the excess of cost which depends on the level of CSR undertaken by the firm (Ni et al. 2010; Manasakis et al. 2013). A different point of view is to consider that CRS efforts induces no additional cost to the firms. In this approach, as a way to incorporate the social goal to the strategic model, a share of consumer's surplus is introduced into the utility function of the social firm (Goering 2007, Lambertini and Tampieri 2010, Kopel and Brand 2011).

The model analyzed in this paper is located in this last approach. We consider a mixed duopoly in which the social firm internalizes its own share of externality and is sensitive to consumer surplus in a decisional context in which both firms face an uncertain demand.

Specifically, we address situations where a profit-maximizing firm competes against a socially responsible firm in a linear homogenous-product duopoly. In contrast to the profit-maximizing firm, the social responsible firm takes into account not only its profits but also a share of consumer surplus. One important difference with the above mentioned papers is that in our model the utility of the social firm is represented by a bi-objective function.

In addition, we introduce demand uncertainty into the model. In the literature on mixed oligopoly we find some papers in which this issue is considered. Thus, Lu and Poddar (2006) analyze a two-stage capacity choice game in mixed duopoly under demand uncertainty, where the firms simultaneously choose the output to produce in stage 2, after the resolution of uncertainty. Anam et al. (2007) analyze demand uncertainty in a mixed oligopoly model in a stochastic environment. They assume that uncertainty is resolved after the leader's commitment to output, but before the follower firm takes its output decision.

In our model, uncertainty in the demand is originated from the fact that different future scenarios are possible and the firms have to make their strategic decisions before uncertainty is resolved. However, when an additional social objective is present there may be other sources of uncertainty. For instance, consider a setting in which consumers could show two different behaviors, social responsible and non-social responsible behavior. If consumers know that the firm is socially responsible, they might be willing to pay a higher reservation price. If the firm is not socially responsible, the reservation price is lower and the market is bigger, given that lower prices could attract more consumers. Assuming linear demand functions we could say that the demand function for socially responsible behavior presents both higher intercept and slope in absolute values than the demand function for non-social responsible behavior. The problem in a mixed duopoly is that consumers face different kinds of firms which in turn implies that firms do not know which kind of consumers they are going to find. Therefore, firms face demand uncertainty.

In order to perform our study, we take as a starting point the results presented in Caraballo et al. (2014), where a Cournot duopoly under demand uncertainty is analyzed. We show that the consideration of a social objective modeled as a function which is increasing with respect to the total quantity in the market, yields new equilibria from those obtained for profit-maximizing firms. In the present paper we investigate the case in which social and non-social firms must decide the quantity to produce before resolving demand uncertainty. In this decision context, the equilibria to which the firms will eventually arrive depend on the firms attitude to risk. We present an analysis of the equilibria for the various cases when one of the firms incorporates the social responsibility objective.

The conclusion is that when a firm incorporates a social objective, new equilibria can emerge. In all of them, irrespectively of the firm's attitude to risk, the socially responsible firm offers quantities greater than or equal to those offered if the firm were a pure profit maximizer.

The rest of the paper is organized as follows. In Section 2 the concept of equilibria when firms value several objectives simultaneously is established. In Section 3 we present our model of mixed duopoly under uncertainty in which one of the firms is a pure profit maximizer and the other incorporates a social objective. The equilibria to which the firms will eventually arrive depending on their attitude towards risk are identified in the various cases. The appendix contains a review of the results in Caraballo et al. (2014) about the equilibria for pure profit maximizers firms which exhibit the same risk attitude, together with the analysis of the cases of pure profit maximizers firms that exhibit different risk attitudes.

2 Pareto equilibria with vector-valued utilities

We consider a two-person normal-form game with vector-valued utility functions, $G = \{(A^i, u^i)_{i=1,2}\}$, where A^i is the set of strategies that agent i can adopt and u^i is a mapping $u^i : A^1 \times A^2 \to \mathbb{R}^{m_i}$, the vector-valued utility function of agent i.

We adopt the term Pareto Equilibrium (PE) to refer to the natural extension of the concept of Nash equilibrium for these games with vector-valued utilities.

Definition 2.1. (q^{*1}, q^{*2}) is a Pareto Equilibrium for the game $G = \{(A^i, u^i)_{i=1,2}\}$ if $\nexists\ q^1 \in A^1$ such that $u^1(q^1, q^{*2}) \geq u^1(q^{*1}, q^{*2})$ with $u^1(q^1, q^{*2}) \neq u^1(q^{*1}, q^{*2})$, and $\nexists\ q^2 \in A^2$ such that $u^2(q^{*1}, q^2) \geq u^2(q^{*1}, q^{*2})$ with $u^2(q^{*1}, q^2) \neq u^2(q^{*1}, q^{*2})$.

The set of Pareto Equilibria for $G = \{(A^i, u^i)_{i=1,2}\}$ is denoted as $PE(G)$.

For $i, j = 1, 2$ with $i \neq j$, denote by R^i the correspondence which represents the best response of agent i to the actions of agent j. In the case of vector-valued utilities, the best response of one agent to an action of the other agent is not in general a singleton, but a subset of its set of strategies, $R^i(q^j) \subseteq A^i$: those strategies of agent i, such that he does not improve his vector-valued utility by deviating from them. A pair of strategies (q^{*1}, q^{*2}) is a Pareto Equilibrium for the game $G = \{(A^i, u^i)_{i=1,2}\}$ if and only if $q^{*i} \in R^i(q^{*j})$ for $i, j = 1, 2$, $i \neq j$.

In the games we investigate in this paper the strategies refer to quantities, thus $A^i \subseteq \mathbb{R}_+$. Moreover, it is assumed that the total quantity the agents are able to offer is bounded by a positive constant, that is $A_i = [0, K^i]$ for $i = 1, 2$.

Example 2.2. As a first example, consider Firm 1 and Firm 2 as profit maximizers which initially compete under Cournot assumptions. They face a linear demand function $p = \alpha - \gamma q$, with α, $\gamma > 0$, have no fixed costs and their marginal costs are equal to zero. In the Cournot game the profit maximizing objectives of the firms are represented by $u^i(q^1, q^2) = q^i(\alpha - \gamma(q^1 + q^2))$, $i = 1, 2$, and the pair of strategies at equilibrium is $(q^{*1}, q^{*2}) = (\frac{\alpha}{3\gamma}, \frac{\alpha}{3\gamma})$.

The case we want to analyze is when Firm 1 together with its profit maximizing objective incorporates a social objective represented by $u^1_2(q^1, q^2) = s(q^1 + q^2)$, where

s is an strictly increasing function in the total quantity, $q = q^1 + q^2$, up to a certain value of q. It is assumed that Firm 1 takes into account the social objective as far as positive profits are obtained. Therefore, the maximum value that q can attain coincides with the market perfect competition quantity, that is, $q = \frac{\alpha}{\gamma}$. Above this quantity, we can assign a negative value to the social objective, for instance $s(q) = -1$.

The shaded area in Figure 1 represents the best responses of Firm 1 to the actions of Firm 2. The perfect competition quantity is denoted by q_{pc}. The Cournot equilibrium quantities when both firms are profit maximizers are q^{jc} and q^{ic} respectively. Observe that when Firm 2 offers q^2, Firm 1 can offer any quantity between his best response in the Cournot game and the quantity which makes the total equal to the perfect competition quantity, since by deviating from these strategies, Firm 1 will always improve one of its objectives an worsen the other. On the other hand, the best response of Firm 2 to the actions of Firm 1 coincides with that of the Cournot game. As a consequence, the set of Pareto equilibria of the extended game is the intersection represented by the dark segment.

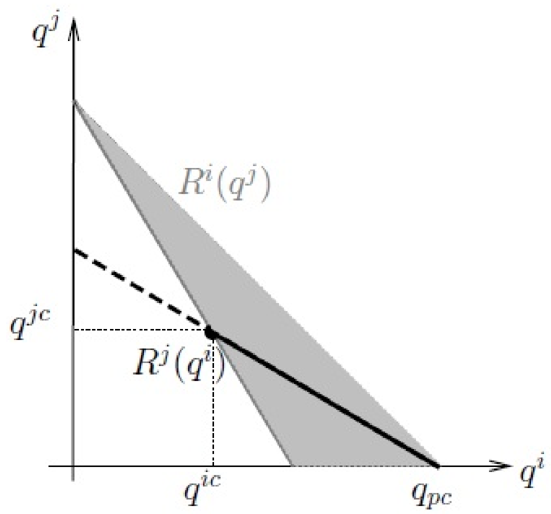

Figure 1. Best reponses and Pareto equilibria.

$$PE(G) = \left\{ (q^1, q^2) : \frac{\alpha}{3\gamma} < q^1 < \frac{\alpha}{\gamma},\, q^2 = \frac{\alpha - \gamma q^1}{2\gamma} \right\}.$$

That is to say, the effect of the incorporation of the social objective is that new equilibria emerge in which the social responsible firm offers quantities greater that its Cournot quantity and the pure profit maximizer firm acts with its best response à la Cournot.

3 Mixed duopoly under uncertainty and attitudes to risk

In a mixed duopoly two firms with different goals are considered. In the model we investigate, one of the firms pursues a social objective in addition to profit maximization, and the other firm is a pure profit maximizer.

Our model of mixed duopoly is the following: two firms producing homogeneous commodities compete in quantities and face uncertain market demand since two different future scenarios are possible. For simplicity we assume that they have no fixed costs and their marginal costs are equal to zero.

The inverse demand function at scenario k, $k = 1, 2$, is given by $p = \alpha_k - \gamma_k k q$, with α_k, $\gamma_k > 0$. In our setting, firms make their output decision, q^1, q^2, before the uncertainty is resolved. For $i = 1, 2$, the benefit for firm i at scenario k is

$$\Pi_k^i(q^1, q^2) = q^i(\alpha_k - \gamma_k(q^1 + q^2)).$$

Without loss of generality, it is assumed that $\frac{\alpha_1}{\gamma_1} < \frac{\alpha_2}{\gamma_2}$, that is, the quantity of perfect competition in the first scenario is lower than that of the second scenario.

One of the firms, say Firm 1, in addition to profit maximization pursues a social objective, whose valuation increases with the total quantity in the market, $q = q^1 + q^2$. A social responsibility objective is often modeled by means of a percentage of the social consumers surplus, hence it increases with the square of the total quantity. The results we present herein hold, provided that the social objective function is increasing in the total quantity offered up to a certain value.

Hence, we represent this social objective function as $u(q^1, q^2) = s(q^1+q^2)$, where s is strictly increasing in the total quantity, $q = q^1 + q^2$, up to a certain value of q.

We assume that Firm 1 values the social objective as long as profits are positive. Otherwise, that is, when the possibility of no making profits at some of the possible scenarios exists, the firm does not take into account the social objective. Since, both firms insure nonnegative profits in both scenarios for quantities below the perfect competition quantity, $q = \frac{\alpha_1}{\gamma_1}$, we can formalize this fact by setting $u(q^1, q^2) = s(q^1 + q^2)$ when $q^1 + q^2 \leq \frac{\alpha_1}{\gamma_1}$, and $u(q^1, q^2) = -1$ when $q^1 + q^2 > \frac{\alpha_1}{\gamma_1}$.

In the present paper we investigate the case in which social and non-social firms must decide the quantity to produce before resolving demand uncertainty. In this decision context, the equilibria to which the firms will eventually arrive depend on the firms attitude to risk. In what follows we present an analysis of the equilibria for the various cases when one of the firms incorporates the social responsibility objective. Interestingly, in some of the cases no equilibria exist for the profit maximizing game. However, the incorporation of an objective reflecting social responsibility may have as a consequence the existence of equilibria strategies.

For the sake of simplicity in the presentation, from all cases for profit maximizers studied in Caraballo et al. (2014), we select those that fulfill the following assumptions.

1. $\frac{2\alpha_2}{3\gamma_2} < \frac{\alpha_1}{\gamma_1}$.
2. $\frac{2\alpha_1}{3\gamma_1} < \frac{\alpha_1-\alpha_2}{\gamma_1-\gamma_2} < \frac{1}{3}(\frac{\alpha_1}{\gamma_1} + \frac{\alpha_2}{\gamma_2})$.

3.1 Firms with identical risk attitude

3.1.1 Conservative firms

In the case in which both firms are conservative, that is, when they exhibit extreme risk aversion, the utility of the firms related to the benefits is represented by the worst benefit obtained in the scenarios. Accordingly, in an equilibrium of the pure profit maximizing game, conservative firms obtain quantities such that no individual deviation produces an improvement in the minimum benefit.

In the mixed duopoly model the vector-valued utility function for Firm 1 is: $u^1 = (u^1_c, u)$ where $u^1_c(q^1, q^2) = Min\{\Pi^1_1(q^1, q^2), \Pi^1_2(q^1, q^2)\}$ and $u(q^1, q^2) = s(q^1+q^2)$.

The real-valued utiliy of Firm 2 is $u^2_c(q^1, q^2) = Min\{\Pi^2_1(q^1, q^2), \Pi^2_2(q^1, q^2)\}$.

Given a game with vector-valued utilities, $G = \{(A^i, u^i)_{i=1,2}\}$, the reaction set of agent i, $R(i)$, contains the pairs of strategies formed by all actions of agent j and the corresponding best responses of agent i. Thus, the reaction set for a conservative Firm 1 which values both the social and the profit maximizing objective, $R(1) = \{(R^1(q^2), q^2) : q^2 \in A^2\}$, is described as:

$$R(1) = \left\{(q^1, q^2) \in \mathbb{R}^2_+ : \frac{\alpha_1 - \alpha_2}{\gamma_1 - \gamma_2} \leq q^1 + q^2 \leq \frac{\alpha_1}{\gamma_1}\right\} \cap \left\{(q^1, q^2) \in \mathbb{R}^2_+ : 2q^1 + q^2 \geq \alpha_1\right\}.$$

The shaded area in the Figure 2 represents this set. The best response of the conservative pure profit maximizer Firm 2 to the actions of Firm 1 can be seen in the

Appendix (Subsection 1.1). As a consequence, the pair of strategies on the broken black line are the Pareto equilibria of this mixed duopoly. Thus, the set of equilibria in this case is:

$$PE(G) = \left\{(q^1, q^2) \in \mathbb{R}^2_+ \,:\, q^1 + q^2 = \frac{\alpha_1 - \alpha_2}{\gamma_1 - \gamma_2}, \, \frac{\alpha_2\gamma_1 - \alpha_1\gamma_2}{\gamma_1(\gamma_1 - \gamma_2)} < q^1 < \frac{\alpha_1\gamma_1 - 2\alpha_2\gamma_1 + \alpha_1\gamma_2}{\gamma_1(\gamma_1 - \gamma_2)}\right\}$$
$$\cup \left\{(q^1, q^2) \in \mathbb{R}^2_+ \,:\, q^2 = \frac{\alpha_1 - \gamma_1 q^1}{2\gamma_1}, \frac{\alpha_1\gamma_1 - 2\alpha_2\gamma_1 + \alpha_1\gamma_2}{\gamma_1(\gamma_1 - \gamma_2)} < q^1 < \frac{\alpha_1}{\gamma_1}.\right\}.$$

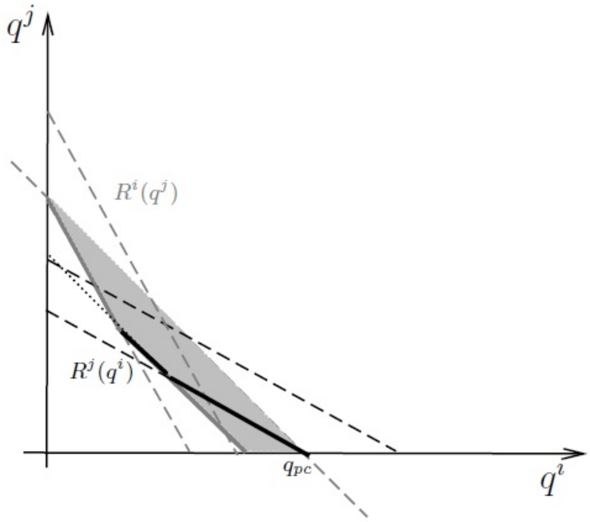

Figure 2. Equilibria for conservative firms.

3.1.2 Optimistic firms

The other extreme case in terms of risk attitude of the firms is the situation when the two firms take into account only the best of the results they can obtain with regard to profits. The utility of optimistic firms is now given by:

$$u^i_{op}(q^1, q^2) = Max\{\Pi^i_1(q^1, q^2), \Pi^i_2(q^1, q^2)\}.$$

This optimistic utility function coincides with Π^i_1 when $(\gamma_1 - \gamma_2)(q^1 + q^2) \leq \alpha_1 - \alpha_2$, and with Π^i_2 otherwise.

The reaction set of an optimistic Firm 1 can be defined as follows

$$R(1) = \left\{(q^1, q^2) \in \mathbb{R}^2_+ \,:\, q^1 + q^2 \leq \frac{\alpha_1}{\gamma_1}\right\} \cap \left\{(q^1, q^2) \in \mathbb{R}^2_+ : 2q^1 + q^2 \geq \alpha_1, q^2 \leq q_m\right\}$$
$$\cap\{(q^1, q^2) \in \mathbb{R}^2_+ \,:\, 2q^1 + q^2 \geq \alpha_2, q^2 \geq q_m\}.$$

where

$$q_m = \frac{\alpha_1 - \alpha_2}{\gamma_1 - \gamma_2} - \frac{1}{\sqrt{\gamma_1\gamma_2}} \frac{\alpha_2\gamma_1 - \alpha_1\gamma_2}{\gamma_1 - \gamma_2}.$$

Regarding the set of equilibria of the mixed duopoly, three cases have to be considered, which depend on the relative position of q_m.

a) For $q_m < \frac{\alpha_1}{3\gamma_1}$ the equilibria are:

$$PE(G) = \left\{ (q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{\alpha_2 - \gamma_2 q^1}{2\gamma_2},\ \frac{\alpha_2}{3\gamma_2} < q^1 < \frac{2\alpha_1}{\gamma_1} - \frac{\alpha_2}{\gamma_2} \right\}$$

b) For $\frac{\alpha_1}{3\gamma_1} < q_m < \frac{\alpha_2}{3\gamma_2}$, the set of equilibria is:

$$PE(G) = \left\{ (q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{\alpha_1 - \gamma_1 q^1}{2\gamma_1},\ \frac{\alpha_1}{3\gamma_1} < q^1 < q_m \right\} \cup$$
$$\left\{ (q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{\alpha_2 - \gamma_2 q^1}{2\gamma_2},\ \frac{\alpha_2}{3\gamma_2} < q^1 < \frac{2\alpha_1}{\gamma_1} - \frac{\alpha_2}{\gamma_2} \right\}.$$

c) For $q_m > \frac{\alpha_2}{3\gamma_2}$, the set of equilibria is:

$$PE(G) = \left\{ (q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{\alpha_1 - \gamma_1 q^1}{2\gamma_1},\ \frac{\alpha_1}{3\gamma_1} < q^1 < q_m \right\} \cup$$
$$\left\{ (q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{\alpha_2 - \gamma_2 q^1}{2\gamma_2},\ q_m < q^1 < \frac{2\alpha_1}{\gamma_1} - \frac{\alpha_2}{\gamma_2} \right\}.$$

Case b) is represented in Figure 3. Note that for these values of the parameters the equilibria in the pure profit maximizing game consist of the two Cournot equilibria. With the new social objective the set of Pareto equilibria is expanded to those pairs of strategies shown in the figure in solid black.

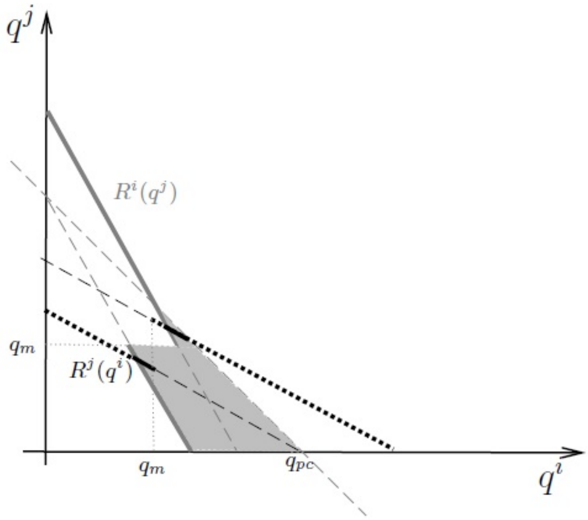

Figure 3. Equilibria for optimistic firms.

3.2 Firms with different risk attitudes

When the attitude towards risk of both firms is different we have to distinguish the following two cases:

1. Firm 1 (the socially responsible) is conservative, and Firm 2 (pure profit maximizer) is optimistic. Three subcases can be distinguished. The first two correspond to those in which equilibria always exist when firms are pure profit maximizer, while in the third case no equilibrium exists for pure profit maximizer duopolists (see appendix).

   a) If $q_m \leq \frac{2}{3}\frac{\alpha_1}{\gamma_1} - \frac{1}{3}\frac{\alpha_2}{\gamma_2}$, the set of equilibria is:

$$PE(G) = \left\{ (q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{\alpha_2 - \gamma_2 q^1}{2\gamma_2},\ \frac{2}{3}\frac{\alpha_1}{\gamma_1} - \frac{1}{3}\frac{\alpha_2}{\gamma_2} < q^1 < \frac{2\alpha_1}{\gamma_1} - \frac{\alpha_2}{\gamma_2} \right\}.$$

   b) If $q_m \geq \left(2(\frac{\alpha_1-\alpha_2}{\gamma_1-\gamma_2}) - \frac{\alpha_1}{\gamma_1}\right) \geq (\frac{2}{3}\frac{\alpha_1}{\gamma_1} - \frac{1}{3}\frac{\alpha_2}{\gamma_2})$, the set of equilibria is given by:

$$PE(G) = \left\{ (q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{\alpha_1 - \gamma_1 q^1}{2\gamma_1},\ 2(\frac{\alpha_1 - \alpha_2}{\gamma_1 - \gamma_2}) - \frac{\alpha_1}{\gamma_1} < q^1 < q_m \right\} \cup$$

$$\left\{ (q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{\alpha_2 - \gamma_2 q^1}{2\gamma_2},\ q_m < q^1 < \frac{2\alpha_1}{\gamma_1} - \frac{\alpha_2}{\gamma_2} \right\}.$$

   c) When $2(\frac{\alpha_1-\alpha_2}{\gamma_1-\gamma_2}) - \frac{\alpha_1}{\gamma_1} > q_m > (\frac{2}{3}\frac{\alpha_1}{\gamma_1} - \frac{1}{3}\frac{\alpha_2}{\gamma_2})$ there is no equilibrium for pure maximizers firms. However, when the new objective is considered, Pareto equilibria may exist for certain values of the parameters. If $q_m \leq \frac{2\alpha_1}{\gamma_1} - \frac{\alpha_2}{\gamma_2}$ the set of equilibria is:

$$PE(G) = \left\{ (q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{\alpha_2 - \gamma_2 q^1}{2\gamma_2},\ q_m < q^1 < \frac{2\alpha_1}{\gamma_1} - \frac{\alpha_2}{\gamma_2} \right\}.$$

   Obviously, if $q_m > \frac{2\alpha_1}{\gamma_1} - \frac{\alpha_2}{\gamma_2}$ no equilibrium exists. Figure 4 represents the two different situations which can occur in this case. On the left-hand side, the equilibria the firms can attain belong to a segment. On the right-hand side no equilibrium exists.

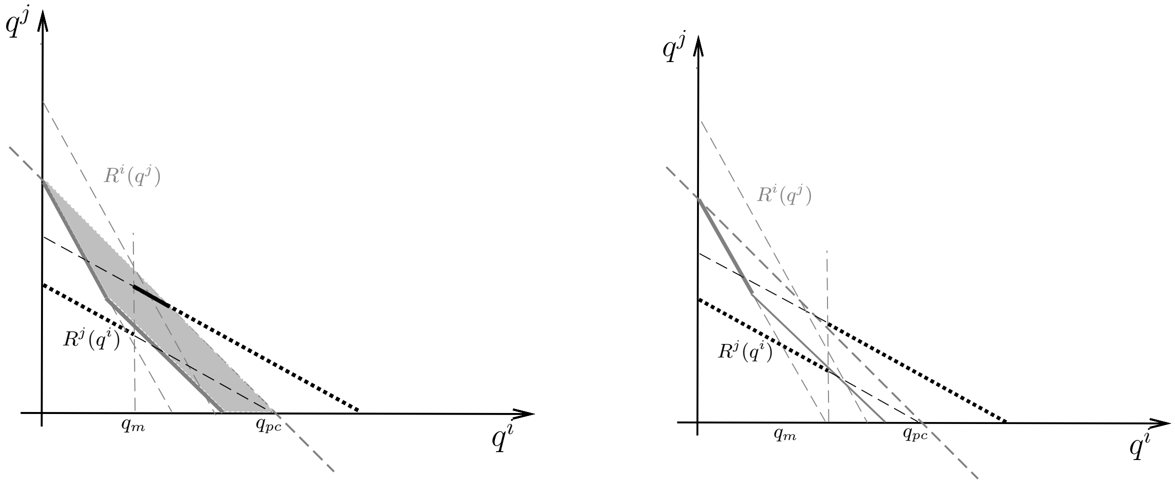

Figure 4. Firm 1 is conservative, Firm 2 is optimistic.

2. In this case, Firm 1 is optimistic and Firm 2 is conservative. We can distinguish the same situations as in case 1.

   a) If $q_m \leq \frac{2}{3}\frac{\alpha_1}{\gamma_1} - \frac{1}{3}\frac{\alpha_2}{\gamma_2}$, the set of equilibria is:

$$PE(G) = \left\{ (q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{\alpha_1 - \gamma_1 q^1}{2\gamma_1},\ \frac{2}{3}\frac{\alpha_2}{\gamma_2} - \frac{1}{3}\frac{\alpha_1}{\gamma_1} < q^1 < \frac{\alpha_1}{\gamma_1} \right\}.$$

   b) If $q_m \geq \left(2(\frac{\alpha_1-\alpha_2}{\gamma_1-\gamma_2}) - \frac{\alpha_1}{\gamma_1}\right) \geq (\frac{2}{3}\frac{\alpha_1}{\gamma_1} - \frac{1}{3}\frac{\alpha_2}{\gamma_2})$, the set of equilibria is given by:

$$PE(G) = \left\{ (q^1, q^2) \in \mathbb{R}^2_+ : q^1 + q^2 = \frac{\alpha_1 - \alpha_2}{\gamma_1 - \gamma_2},\ \frac{\alpha_1}{\gamma_1} - \frac{\alpha_1 - \alpha_2}{\gamma_1 - \gamma_2} < q^1 < 2\frac{\alpha_1 - \alpha_2}{\gamma_1 - \gamma_2} - \frac{\alpha_1}{\gamma_1} \right\}$$

$$\cup \left\{ (q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{\alpha_1 - \gamma_1 q^1}{2\gamma_1},\ 2\frac{\alpha_1 - \alpha_2}{\gamma_1 - \gamma_2} - \frac{\alpha_1}{\gamma_1} < q^1 < \frac{\alpha_1}{\gamma_1} \right\}.$$

   c) When $2(\frac{\alpha_1-\alpha_2}{\gamma_1-\gamma_2}) - \frac{\alpha_1}{\gamma_1} > q_m > (\frac{2}{3}\frac{\alpha_1}{\gamma_1} - \frac{1}{3}\frac{\alpha_2}{\gamma_2})$, no equilibrium for pure maximizers firms exists. In this case, unlike the situation where both firms are profit maximizers or the social firm is conservative and the profit maximizer is optimistic, it can be assured that equilibria always exist and the set of equilibria can be described as follows:

$$PE(G) = \left\{ (q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{\alpha_1 - \gamma_1 q^1}{2\gamma_1},\ 2\frac{\alpha_1 - \alpha_2}{\gamma_1 - \gamma_2} - \frac{\alpha_1}{\gamma_1} < q^1 < \frac{\alpha_1}{\gamma_1} \right\} \cup$$

$$\left\{ (q^1, q^2) \in \mathbb{R}^2_+ : q^1 + q^2 = \frac{\alpha_1 - \alpha_2}{\gamma_1 - \gamma_2},\ \frac{1}{\sqrt{\gamma_1 \gamma_2}} \frac{\alpha_2 \gamma_1 - \alpha_1 \gamma_2}{\gamma_1 - \gamma_2} < q^1 < 2\frac{\alpha_1 - \alpha_2}{\gamma_1 - \gamma_2} - \frac{\alpha_1}{\gamma_1} \right\}.$$

This set is represented in Figure 5.

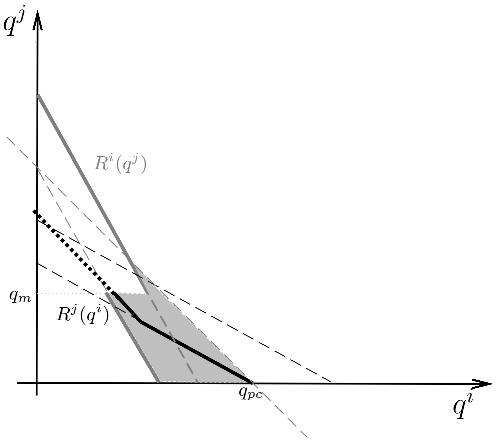

Figure 5. Equilibria for optimistic Firm 1, conservative Firm 2.

Example 3.1. Consider the Cournot game under uncertainty in which the demand functions at scenario 1 and 2 are respectively: $p = 10 - q$ and $p = 5 - (7/20)q$. In this case $\alpha_1 = 10$, $\gamma_1 = 1$, $\alpha_2 = 5$, $\gamma_2 = 7/20$. Following Theorem 3.9.c) in Caraballo et al. (2014), since the quantity $q_m = \frac{100\sqrt{7/20}-30}{13\sqrt{7/20}}$ is located between the Cournot equilibria of both markets, the optimistic equilibria are the Cournot equilibrium of each market: $(10/3, 10/3)$ and $(100/21, 100/21)$. The conservative equilibria are those Pareto equilibria (q^1, q^2), such that $q^1 + q^2 = 100/13$. In the set of conservative equilibria, the quantity each firm produces varies from 30/13 to 70/13.

Since $q_m = \frac{100\sqrt{7/20}-30}{13\sqrt{7/20}}$, $\frac{\alpha_1-\alpha_2}{\gamma_1-\gamma_2} = \frac{100}{13}$, $\frac{1}{3}(\frac{\alpha_1}{\gamma_1} + \frac{\alpha_2}{\gamma_2}) = \frac{170}{21}$, $(\frac{2}{3}\frac{\alpha_1}{\gamma_1} - \frac{1}{3}\frac{\alpha_2}{\gamma_2}) = \frac{40}{21}$ and $2(\frac{\alpha_1-\alpha_2}{\gamma_1-\gamma_2}) - \frac{\alpha_1}{\gamma_1} = \frac{70}{13}$, this example corresponds to case c) in which, when firms show different attitudes to risk, no equilibrium exists.

If Firm 1 has a second objective and Firm 2 is a profit maximizer, we distinguish the following cases

1. The case in which both firms are conservative corresponds to a situation as represented in Figure 2. The set of PE is:

$$PE(G) = \left\{(q^1, q^2) \in \mathbb{R}^2_+ : q^1 + q^2 = \frac{100}{13},\ \frac{30}{13} < q^1 < \frac{70}{13}\right\} \cup$$

$$\left\{(q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{10 - q^1}{2},\ \frac{70}{13} < q^1 < 10\right\}.$$

2. The situation in which both firms are optimistic is represented in Figure 3. The set of PE is:

$$PE(G) = \left\{(q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{10 - q^1}{2},\ \frac{10}{30} < q^1 < q_m\right\} \cup$$

$$\left\{(q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{50}{7} - \frac{q^1}{2},\ \frac{100}{21} < q^1 < \frac{40}{7}\right\}.$$

3. If Firm 1 is conservative and Firm 2 is optimistic, as in Figure 4 (left), the set of PE is:

$$PE(G) = \left\{(q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{50}{7} - \frac{q^1}{2},\ q_m < q^1 < \frac{40}{7}\right\}.$$

4. When Firm 1 is optimistic and Firm 2 is conservative as in Figure 5, the set of PE is:

$$PE(G) = \left\{(q^1, q^2) \in \mathbb{R}^2_+ : q^2 = \frac{10 - q^1}{2},\ \frac{70}{13} < q^1 < 10\right\} \cup$$

$$\left\{(q^1, q^2) \in \mathbb{R}^2_+ : q^1 + q^2 = \frac{100}{13},\ \sqrt{\frac{20}{7}}\frac{30}{13} < q^1 < \frac{70}{13}\right\}.$$

4 Conclusions

An alternative analysis of the effect of strategic corporate social responsibility in a mixed duopoly under demand uncertainty, which differs from those of existing in the literature, is presented. In our model, the social firm faces a bi-objective utility function, which reflects profit maximizing under uncertainty, together with the pursuit of a social goal. We have shown that the set of equilibria of the mixed duopoly expands the equilibria of the profit maximizer strategic model. In all the new equilibria which emerge, irrespectively of the firm attitude to risk, the socially responsible firm offers quantities greater than or equal to those offered in the classic pure profit maximizer duopoly.

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5 Appendix: Equilibria for profit maximizers under uncertainty and risk attitudes

In this appendix we summarize some results extracted from Caraballo et al. (2014) and we present the analysis of the equilibria in new situations. In Caraballo et al. a normal form game with vector-valued utility functions is considered in order to analyse a Cournot duopoly under demand uncertainty in a context in which two future scenarios are possible. The inverse demand function at scenario k, $k = 1, 2$, is given by $p = \alpha_k - \gamma_k q$, with α_k, $\gamma_k > 0$. It is assumed that firms have no fixed costs and their marginal costs are equal to zero. The firms make their output decision, q^1, q^2, before the uncertainty is resolved.

For $i, j = 1, 2$ with $i \neq j$, denote $r^i_k : A_j \to \mathbb{R}$ as the function which represents the best response of agent i to the actions of agent j at scenario k,

$$r^i_k(q^j) = \frac{\alpha_k - \gamma_k q^j}{2\gamma_k}.$$

We next present the reaction functions and the set of equilibria when firms show extreme attitudes to risk and the parameters of the demand function fulfill the following assumptions:

1. $\frac{\alpha_2}{2\gamma_2} < \frac{\alpha_1}{\gamma_1}$.

2. $\frac{2\alpha_1}{3\gamma_1} < \frac{\alpha_1-\alpha_2}{\gamma_1-\gamma_2} < \frac{1}{3}\left(\frac{\alpha_1}{\gamma_1} + \frac{\alpha_2}{\gamma_2}\right)$.

Assumption 1 implies that the set of equilibria of the Cournot game under uncertainty when both firms are profit maximizers are positive strategies. The second assumption implies that the intersection of the demand functions is between the Cournot equilibrium of market 1 and the equilibrium quantity when firms assume different demand function when taking their decisions.

5.1 Firms with identical risk attitude

Conservative firms

In this case the reaction function for Firm 1 (symmetrically for Firm 2) is defined as follows:

$$r_k^1(q^2) = \begin{cases} \dfrac{\alpha_1 - \alpha_2}{\gamma_1 - \gamma_2} - q^2 & \text{if } q^2 < \frac{\alpha_1-\alpha_2}{\gamma_1-\gamma_2} - q^1, \\ \dfrac{\alpha_1 - \gamma_1 q^2}{2\gamma_1} & \text{otherwise.} \end{cases}$$

And the set of equilibria is given by:

$$E^c = \left\{ (q^1, q^2) : \, q^1 + q^2 = \frac{\alpha_1 - \alpha_2}{\gamma_1 - \gamma_2}, \; \frac{\alpha_2\gamma_1 - \alpha_1\gamma_2}{\gamma_1(\gamma_1 - \gamma_2)} < q^1 < \frac{\alpha_1\gamma_1 - 2\alpha_2\gamma_1 + \alpha_1\gamma_2}{\gamma_1(\gamma_1 - \gamma_2)} \right\}.$$

Figure 6 shows the reaction functions for both firms and the set of equilibria. The dashed line corresponds to firm 1 and the dotted line corresponds to Firm 2.

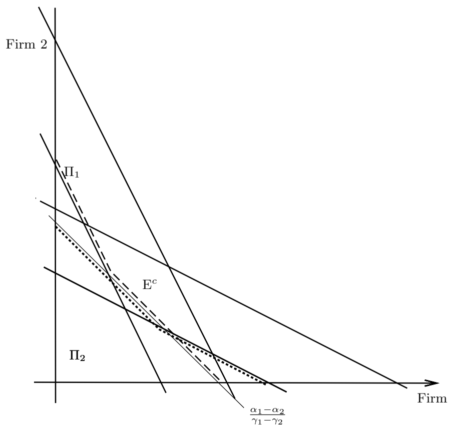

Figure 6. Equilibria for conservative profit maximizer firms.

Optimistic firms

In this case the reaction function for Firm 1 (symmetrically for Firm 2) is defined as follows:

$$r_k^1(q^1) = \begin{cases} \dfrac{\alpha_1 - \gamma_1 q^2}{2\gamma_1} & \text{if } q_2 \leq q_m \\ \dfrac{\alpha_2 - \gamma_2 q^2}{2\gamma_2} & \text{otherwise.} \end{cases}$$

where $q_m = \frac{\alpha_1-\alpha_2}{\gamma_1-\gamma_2} - \frac{1}{\sqrt{\gamma_1\gamma_2}}\frac{\alpha_2\gamma_1-\alpha_1\gamma_2}{\gamma_1-\gamma_2}$.

The possible equilibria reduce to the Cournot strategies in both scenarios. Depending of the values of the parameters, there may exist an unique optimistic equilibrium which coincides with the Cournot equilibrium of one of the scenarios, or both Cournot equilibria of the scenarios are optimistic equilibria.

$$E^{op} \subseteq \left\{ \left(\frac{\alpha_1}{3\gamma_1}, \frac{\alpha_1}{3\gamma_1}\right), \left(\frac{\alpha_2}{3\gamma_2}, \frac{\alpha_2}{3\gamma_2}\right) \right\}.$$

Figure 7 shows the reaction functions for both firms and the set of equilibria when q_m is below the Cournot equilibrium of market 1 (left) and when q_m is between the Cournot equilibrium of markets 1 and 2 (right) [1]. The dashed line corresponds to Firm 1 and the dotted line corresponds to Firm 2.

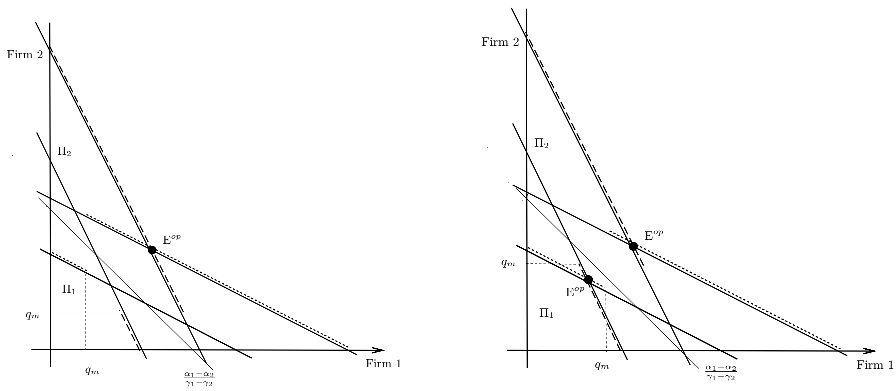

Figure 7. Equilibria for optimistic profit maximizer firms.

5.2 Firms with different risk attitudes

In addition to the results in Caraballo et al.(2014), we present now the analysis of the case where firms show different attitudes to risk. Let us consider that Firm 1 is conservative and Firm 2 is optimistic. In order to obtain the set of equilibria we take into account the corresponding reaction functions.

1. If $q_m \leq \frac{2}{3}\frac{\alpha_1}{\gamma_1} - \frac{1}{3}\frac{\alpha_2}{\gamma_2}$, then

$$E^{c,op} = \left\{ \left(\frac{2}{3}\frac{\alpha_1}{\gamma_1} - \frac{1}{3}\frac{\alpha_2}{\gamma_2}, \frac{2}{3}\frac{\alpha_2}{\gamma_2} - \frac{1}{3}\frac{\alpha_1}{\gamma_1}\right) \right\}$$

[1] if q_m is above the Cournot equilibrium of market 2, the unique equilibrium will be the Cournot equilibrium of market 1.

2. If $q_m \geq 2(\frac{\alpha_1-\alpha_2}{\gamma_1-\gamma_2}) - \frac{\alpha_1}{\gamma_1} \geq (\frac{2}{3}\frac{\alpha_1}{\gamma_1} - \frac{1}{3}\frac{\alpha_2}{\gamma_2})$, then

$$E^{c,op} = \left\{ \left(2(\frac{\alpha_1-\alpha_2}{\gamma_1-\gamma_2}) - \frac{\alpha_1}{\gamma_1}, \frac{\alpha_1}{\gamma_1} - \frac{\alpha_1-\alpha_2}{\gamma_1-\gamma_2} \right) \right\}$$

Figure 8 shows both cases. The dashed line corresponds to Firm 1 and the dotted line corresponds to Firm 2.

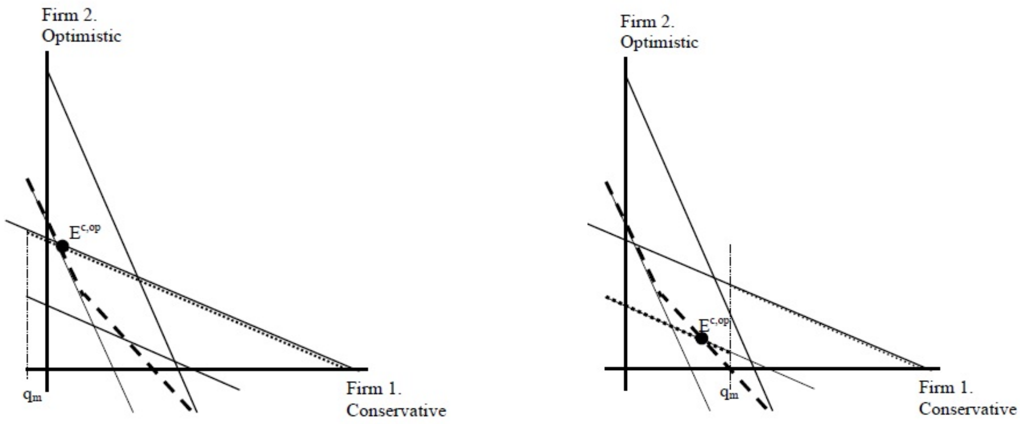

Figure 8. Equilibria for profit maximizers with different attitudes to risk.

Nevertheless, unlike cases where both firms show the same attitude to risk, when they exhibit opposite attitude to risk, there are cases for which no equilibrium exists as represented in Figure 9. This is the case when

$$2\left(\frac{\alpha_1-\alpha_2}{\gamma_1-\gamma_2}\right) - \frac{\alpha_1}{\gamma_1} > q_m > \left(\frac{2}{3}\frac{\alpha_1}{\gamma_1} - \frac{1}{3}\frac{\alpha_2}{\gamma_2}\right).$$

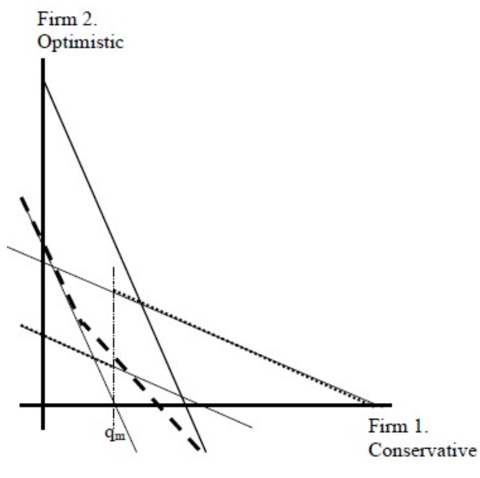

Figure 9. No equilibrium for profit maximizers with different attitudes to risk.

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